Name ____

Algebra 1 Summer Review Packet
Ramapo Indian Hills

This packet must be completed the summer before entering Algebra 1. These skills are necessary for a successful year in this course. Please review the notes for each section and complete all subsequent practice problems. Be sure to show all of your work. This packet must be completed in its entirety and be ready to be submitted on the first day of school.

I. Writing Algebraic Expressions

In **algebraic expressions,** letters such as *x* and *w* are called variables. A variable is used to represent an unspecified number or value.

Practice: Write an algebraic expression for each verbal expression.

1. Four times a number decreased by twelve ____________________
2. Three more than the product of five and a number ________________
3. The quotient of two more than a number and eight ________________
4. Seven less than twice a number __________________________

II. Order of Operations

To evaluate numerical expressions containing more than one operation, use the rules for order of operations. The rules are often summarized using the expression **PEMDAS**

Examples:

Parentheses (Grouping Symbols)	$[(7-4)^2+3]+15$	$\frac{(9-7)^2+6}{11-6}$
Exponents	$=[3^2+3]+15$	$=\frac{2^2+6}{5}$
Multiply or Divide, from left to right	$=[9+3]+15$	$=\frac{4+6}{5}$
Add or Subtract, from left to right	$=12+15$	$=\frac{10}{2}$
		$=5$

Practice: Evaluate each expression.

1. $250 \div [5(3 \bullet 7 + 4)]$

2. $\dfrac{5^2 \cdot 4 - 5 \cdot 4^2}{5(4)}$

3. $\dfrac{1}{2} \cdot 26 - 3^2$

4. $8^2 \div (2 \bullet 8) + 2$

5. $5 + \left[30 - (6-1)^2\right]$

6. $\dfrac{2 \cdot 4^2 - 8 \div 2}{(5+2) \cdot 2}$

III. Evaluating Algebraic Expressions

To evaluate algebraic expressions, first replace the variables with their values. Then, use order of operations to calculate the value of the resulting numerical expression.

Example: Evaluate $x^2 - 5(x - y)$ if $x = 6$ and $y = 2$

$$\begin{aligned} x^2 - 5(x - y) &= (6)^2 - 5(6 - 2) \\ &= (6)^2 - 5(4) \\ &= 36 - 5(4) \\ &= 36 - 20 \\ &= 16 \end{aligned}$$

Practice: Evaluate each expression.

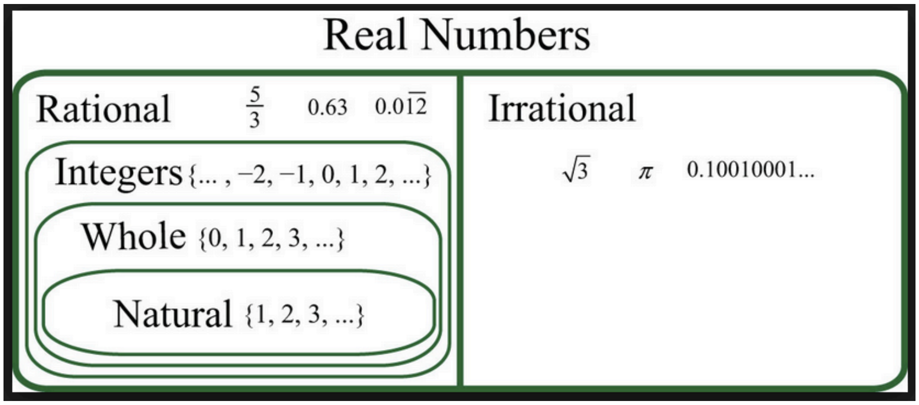
1. $5x^2 - y$ when $x = 4$ and $y = 24$

2. $\frac{3xy - 4}{7x}$ when $x = 2$ and $y = 3$

3. $(z \div x)^2 + \frac{4}{5}x$ when $x = 2$ and $z = 4$

4. $\frac{y^2 - 2z^2}{x + y - z}$ when $x = 12$, $y = 9$, and $z = 4$

IV. The Real Number System



The **Real** number system is made up of two main sub-groups **Rational numbers** and **Irrational numbers.**

The set of rational numbers includes several subsets: **natural numbers, whole numbers, and integers.**

- **Real Numbers-** any number that can be represented on a number-line.
  - **Rational Numbers-** a number that can be written as the ratio of two integers (this includes decimals that have a definite end or repeating pattern)
    Examples: $2, -5, \frac{-3}{2}, \frac{1}{3}, 0.253, 0.\overline{3}$
    - **Integers-** positive and negative whole numbers and 0
      Examples: $-5, -3, 0, 8$ ...
    - **Whole Numbers** – the counting numbers from 0 to infinity
      Examples: { 0, 1, 2, 3, 4, ...}
    - **Natural Numbers-** the counting numbers from 1 to infinity
      Examples: { 1, 2, 3, 4… }
  - **Irrational Numbers-** Non-terminating, non-repeating decimals (including π, and the square root of any number that is not a perfect square.)
    Examples: 2π, $\sqrt{3}$, $\sqrt{23}$, 3.21211211121111....

Practice: Name all the sets to which each number belongs.

1. -4.2 ______________________________
2. $3\sqrt{5}$ ______________________________
3. $\frac{5}{3}$ ______________________________
4. 9 ______________________________
5. $\sqrt{16}$ ______________________________
6. $-\frac{8}{2}$ ______________________________

V. Properties of Real Numbers

Following are properties of Real Numbers that are useful in evaluating and solving algebraic expressions.

Additive Identity	For any number a, $a + 0 = a$.
Multiplicative Identity	For any number a, $a \cdot 1 = a$.
Multiplicative Property of 0	For any number a, $a \cdot 0 = 0$.
Multiplicative Inverse Property	For every number $\frac{a}{b}$, $a, b \neq 0$, there is exactly one number $\frac{b}{a}$ such that $\frac{a}{b} \cdot \frac{b}{a} = 1$.
Reflexive Property	For any number a, $a = a$.
Symmetric Property	For any numbers a and b, if $a = b$, then $b = a$.
Transitive Property	For any numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.
Substitution Property	If $a = b$, then a may be replaced by b in any expression.
Commutative Properties	For any numbers a and b, $a + b = b + a$ and $a \cdot b = b \cdot a$.
Associative Properties	For any numbers a, b, and c, $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$.

Practice: Name the property illustrated in each equation.

1. $3 \bullet x = x \bullet 3$ ____________________________________

2. $3a + 0 = 3a$ ____________________________________

3. $2r + (3r + 4r) = (2r + 3r) + 4r$ ______________________

4. $5y \bullet \frac{1}{5y} = 1$ ____________________________________

5. $9a + (-9a) = 0$ ________________________________

6. $(10b + 12b) + 7b = (12b + 10b) + 7b$ ______________________

7. $5x + 2 = 5x + 2$ ___________________________

8. If $9 + 4 = 13$ and $13 = 2 + 11$ then $9 + 4 = 2 + 11$ ______________________

9. If $x = 7$ then $7 = x$ ___________________

10. $3 \bullet 1 = 3$ ________________________

VI. The Distributive Property

The Distributive Property states for any number *a, b,* and *c*:

1. $a(b+c) = ab + ac$ or $(b+c)a = ba + ca$
2. $a(b-c) = ab - ac$ or $(b-c)a = ba - ca$

Practice: Rewrite each expression using the distributive property.

1. $7(h - 3)$

2. $-3(2x + 5)$

3. $(5x - 9)4$

4. $\frac{1}{2}(14 - 6y)$

5. $3(7x^2 - 3x + 2)$

6. $\frac{1}{4}(16x - 12y + 4z)$

7. $(9 - 2x + 3xy) \cdot -4$

8. $0.3(40a + 10b - 5)$

VII. Combining Like-Terms

Terms in algebra are numbers, variables or the product of numbers and variables. In algebraic expressions terms are separated by addition (+) or subtraction (-) symbols. Terms can be combined using addition and subtraction if they are **like-terms**.

Like-terms have the same variables to the same power.

Example of like-terms: $5x^2$ and $-6x^2$

Example of terms that are **NOT** like-terms: $9x^2$ and $15x$

Although both terms have the variable ***x****, they are not being raised to the same power*

To combine like-terms using addition and subtraction, add or subtract the numerical factor

Example: Simplify the expression by combining like-terms

$$8x^2+9x-12x+7x^2 = (8+7)x^2+(9-12)x$$
$$= 15x^2+-3x$$
$$= 15x^2-3x$$

Practice: Simplify each expression

1. $5x-9x+2$

2. $3q^2+q-q^2$

3. $c^2+4d^2-7d^2$

4. $5x^2+6x-12x^2-9x+2$

5. $2(3x-4y)+5(x+3y)$

6. $10xy-4(xy+2x^2y)$

VIII. Solving Equations with Variables on One-Side

To solve an equation means to ***find the value*** of the variable. We solve equations by isolating the variable using opposite operations.

Example:

Solve.

$3x - 2 = 10$	
$+2 \quad +2$	Isolate 3x by adding 2 to each side.
$\frac{3x}{3} = \frac{12}{3}$	Simplify Isolate x by dividing each side by 3.
$x = 4$	Simplify

Check your answer.

$3(4) - 2 = 10$	Substitute the value in for the variable.
$12 - 2 = 10$	Simplify
$10 = 10$	Is the equation true? If yes, you solved it correctly!

Opposite Operations:
Addition (+) & Subtraction (–)
Multiplication (x) & Division (÷)

Please remember…
to do the same step on each side of the equation.

Always check your work by substitution!

Practice: Solve each equation.

1. $98 = b + 34$

2. $-14 + y = -2$

3. $8k = -64$

4. $\frac{2}{5}x = 6$

5. $14n - 8 = 34$

6. $8 + \frac{n}{12} = 13$

7. $\frac{3k-7}{5} = 16$

8. $-\frac{d}{6} + 12 = -7$

IX. Solving Equations with Variables on Each-Side:

To solve an equation with the same variable on each side, write an equivalent equation that has the variable on just one side of the equation. Then solve.

Example **Solve $4(2a - 1) = -10(a - 5)$.**

$4(2a - 1) = -10(a - 5)$	Original equation
$8a - 4 = -10a + 50$	Distributive Property
$8a - 4 + 10a = -10a + 50 + 10a$	Add 10a to each side.
$18a - 4 = 50$	Simplify.
$18a - 4 + 4 = 50 + 4$	Add 4 to each side.
$18a = 54$	Simplify.
$\frac{18a}{18} = \frac{54}{18}$	Divide each side by 18.
$a = 3$	Simplify.

The solution is 3.

Practice: Solve each equation.

1. $5 + 3r = 5r - 19$

2. $8x + 12 = 4(3 + 2x)$

3. $-5x - 10 = 2 - (x + 4)$

4. $6(-3m + 1) = 5(-2m - 2)$

5. $3(d - 8) - 5 = 9(d + 2) + 1$

X. Ratios and Proportions

Ratios and Proportions A **ratio** is a comparison of two numbers by division. The ratio of x to y can be expressed as x to y, $x{:}y$ or $\frac{x}{y}$. Ratios are usually expressed in simplest form.

An equation stating that two ratios are equal is called a **proportion**. To determine whether two ratios form a proportion, express both ratios in simplest form or check cross products.

Example 1 **Determine whether the ratios $\frac{24}{36}$ and $\frac{12}{18}$ form a proportion.**

$\frac{24}{36} = \frac{2}{3}$ when expressed in simplest form.

$\frac{12}{18} = \frac{2}{3}$ when expressed in simplest form.

The ratios $\frac{24}{36}$ and $\frac{12}{18}$ form a proportion because they are equal when expressed in simplest form.

Example 2 **Use cross products to determine whether $\frac{10}{18}$ and $\frac{25}{45}$ form a proportion.**

$\frac{10}{18} \stackrel{?}{=} \frac{25}{45}$	Write the proportion.
$10(45) \stackrel{?}{=} 18(25)$	Cross products
$450 = 450$	Simplify.

The cross products are equal, so $\frac{10}{18} = \frac{25}{45}$. Since the ratios are equal, they form a proportion.

Practice: Determine whether each pair of ratios forms a proportion.

1. $\frac{12}{32}, \frac{3}{16}$

2. $\frac{15}{20}, \frac{9}{12}$

3. $\frac{1.5}{2}, \frac{6}{8}$

4. 5 to 9, 25 to 45

5. 0.1 to 0.2, 0.45 to 1.35

6. 100:75, 44:33

X. Ratios and Proportions (Continued)

Solve Proportions If a proportion involves a variable, you can use cross products to solve the proportion. In the proportion $\frac{x}{5} = \frac{10}{13}$, x and 13 are called **extremes.** They are the first and last terms of the proportion. 5 and 10 are called **means**. They are the middle terms of the proportion. In a proportion, the product of the extremes is equal to the product of the means.

Means-Extremes Property of Proportions	For any numbers a, b, c, and d, if $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

Example 1:

$$\frac{x}{5} = \frac{10}{13}$$

$$x \cdot 13 = 5 \cdot 10$$

$$13x = 50$$

$$\frac{13x}{13} = \frac{50}{13}$$

$$x = \frac{50}{13}$$

Example 2:

$$\frac{x+1}{4} = \frac{3}{4}$$

$$4(x+1) = 3 \cdot 4$$

$$4x + 4 = 12$$
$$-4 \quad -4$$

$$4x = 8$$

$$\frac{4x}{4} = \frac{8}{4}$$

$$x = 2$$

Practice: Solve each proportion.

1. $\frac{x}{21} = \frac{3}{63}$

2. $\frac{-3}{x} = \frac{2}{8}$

3. $\frac{0.1}{2} = \frac{0.5}{x}$

4. $\frac{9}{y+1} = \frac{18}{54}$

5. $\frac{a-8}{12} = \frac{15}{3}$

6. $\frac{3+y}{4} = \frac{-y}{8}$

XI. Percent of Change

Percent of Change When an increase or decrease in an amount is expressed as a percent, the percent is called the **percent of change**. If the new number is greater than the original number, the percent of change is a **percent of increase**. If the new number is less than the original number, the percent of change is the **percent of decrease**.

Example 1

Find the percent of increase.
original: 48
new: 60

First, subtract to find the amount of increase. The amount of increase is $60 - 48 = 12$.

Then find the percent of increase by using the original number, 48, as the base.

$\frac{12}{48} = \frac{r}{100}$	Percent proportion
$12(100) = 48(r)$	Cross products
$1200 = 48r$	Simplify.
$\frac{1200}{48} = \frac{48r}{48}$	Divide each side by 48.
$25 = r$	Simplify.

The percent of increase is 25%.

Example 2

Find the percent of decrease.
original: 30
new: 22

First, subtract to find the amount of decrease. The amount of decrease is $30 - 22 = 8$.

Then find the percent of decrease by using the original number, 30, as the base.

$\frac{8}{30} = \frac{r}{100}$	Percent proportion
$8(100) = 30(r)$	Cross products
$800 = 30r$	Simplify.
$\frac{800}{30} = \frac{30r}{30}$	Divide each side by 30.
$26\frac{2}{3} = r$	Simplify.

The percent of decrease is $26\frac{2}{3}\%$, or about 27%.

Practice: State whether each percent of change is a percent of increase or percent of decrease. Then find each percent of change.

1. original: 50
   new: 80

3. original: 27.5
   new: 25

2. original: 14.5
   new: 10

4. original 250:
   new: 500

XI. Percent of Change (Continued)

Solve Problems Discounted prices and prices including tax are applications of percent of change. Discount is the amount by which the regular price of an item is reduced. Thus, the discounted price is an example of percent of decrease. Sales tax is amount that is added to the cost of an item, so the price including tax is an example of percent of increase.

Example **A coat is on sale for 25% off the original price. If the original price of the coat is $75, what is the discounted price?**

The discount is 25% of the original price.

25% of $75 = 0.25 × 75   25% = 0.25
= 18.75   Use a calculator.

Subtract $18.75 from the original price.
$75 − $18.75 = $56.25

The discounted price of the coat is $56.25.

Practice: Find the final price of each item. When a discount and sales tax are listed, compute the discount price before computing the tax.

1. Two concert tickets: $28
   Student discount: 28%

2. Airline ticket: $248.00
   Frequent Flyer discount: 33%

3. CD player: $142.00
   Sales tax: 5.5%

4. Celebrity calendar: $10.95
   Sales tax: 7.5%

5. Camera: $110.95
   Discount: 20%
   Sales tax: 5%

6. Ipod: $89.00
   Discount: 17%
   Tax: 5%

XII. Solving For a Specific Variable

Solve for Variables Sometimes you may want to solve an equation such as $V = \ell wh$ for one of its variables. For example, if you know the values of V, w, and h, then the equation $\ell = \frac{V}{wh}$ is more useful for finding the value of ℓ. If an equation that contains more than one variable is to be solved for a specific variable, use the properties of equality to isolate the specified variable on one side of the equation.

Example 1 **Solve $2x - 4y = 8$ for y.**

$$
\begin{aligned}
2x - 4y &= 8 \\
2x - 4y - 2x &= 8 - 2x \\
-4y &= 8 - 2x \\
\frac{-4y}{-4} &= \frac{8 - 2x}{-4} \\
y &= \frac{8 - 2x}{-4} \text{ or } \frac{2x - 8}{4}
\end{aligned}
$$

The value of y is $\frac{2x - 8}{4}$.

Example 2 **Solve $3m - n = km - 8$ for m.**

$$
\begin{aligned}
3m - n &= km - 8 \\
3m - n - km &= km - 8 - km \\
3m - n - km &= -8 \\
3m - n - km + n &= -8 + n \\
3m - km &= -8 + n \\
m(3 - k) &= -8 + n \\
\frac{m(3 - k)}{3 - k} &= \frac{-8 + n}{3 - k} \\
m &= \frac{-8 + n}{3 - k}, \text{ or } \frac{n - 8}{3 - k}
\end{aligned}
$$

The value of m is $\frac{n - 8}{3 - k}$. Since division by 0 is undefined, $3 - k \neq 0$, or $k \neq 3$.

Practice: Solve each equation or formula for the variable specified.

1. $15x + 1 = y$ for x

2. $x(4 - k) = p$ for k

3. $7x + 3y = m$ for y

4. $P = 2l + 2w$ for w

XIII. Rate of Change and Slope

Find Slope

Slope of a Line	$m = \frac{\text{rise}}{\text{run}}$ or $m = \frac{y_2 - y_1}{x_2 - x_1}$, where (x_1, y_1) and (x_2, y_2) are the coordinates of any two points on a nonvertical line

Example 1 **Find the slope of the line that passes through (−3, 5) and (4, −2).**

Let $(-3, 5) = (x_1, y_1)$ and $(4, -2) = (x_2, y_2)$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{-2 - 5}{4 - (-3)}$ $y_2 = -2, y_1 = 5, x_2 = 4, x_1 = -3$

$= \frac{-7}{7}$ Simplify.

$= -1$

Example 2 **Find the value of r so that the line through (10, r) and (3, 4) has a slope of $-\frac{2}{7}$.**

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$-\frac{2}{7} = \frac{4 - r}{3 - 10}$ $m = -\frac{2}{7}, y_2 = 4, y_1 = r, x_2 = 3, x_1 = 10$

$-\frac{2}{7} = \frac{4 - r}{-7}$ Simplify.

$-2(-7) = 7(4 - r)$ Cross multiply.

$14 = 28 - 7r$ Distributive Property

$-14 = -7r$ Subtract 28 from each side.

$2 = r$ Divide each side by −7.

Practice:
Find the slope of the line that passes through each pair of points.

1. $(4,9), (1-,6)$
2. $(2,5), (6,2)$
3. $(4, 3.5), (-4,3.5)$
4. $(1,-2), (-2,-5)$

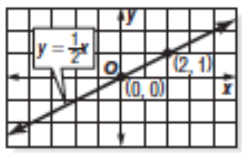
Determine the value of *r* so the line that passes through ach pair of points has the given slope.

5. $(6,8), (r,-2), m = 1$
6. $(10, r), (3,4), m = -\frac{2}{7}$

XIV. Slope and Direct Variation

Direct Variation A **direct variation** is described by an equation of the form $y = kx$, where $k \neq 0$. We say that *y varies directly as x*. In the equation $y = kx$, k is the **constant of variation**.

Example 1 **Name the constant of variation for the equation. Then find the slope of the line that passes through the pair of points.**



For $y = \frac{1}{2}x$, the constant of variation is $\frac{1}{2}$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$= \frac{1 - 0}{2 - 0}$ $(x_1, y_1) = (0, 0), (x_2, y_2) = (2, 1)$

$= \frac{1}{2}$ Simplify.

The slope is $\frac{1}{2}$.

Example 2 **Suppose y varies directly as x, and $y = 30$ when $x = 5$.**

a. Write a direct variation equation that relates x and y.

Find the value of k.

$y = kx$ Direct variation equation

$30 = k(5)$ Replace y with 30 and x with 5.

$6 = k$ Divide each side by 5.

Therefore, the equation is $y = 6x$.

b. Use the direct variation equation to find x when $y = 18$.

$y = 6x$ Direct variation equation

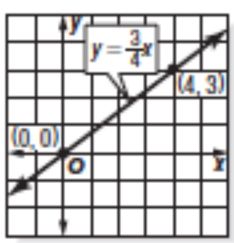
$18 = 6x$ Replace y with 18.

$3 = x$ Divide each side by 6.

Therefore, $x = 3$ when $y = 18$.

Practice:
Name the constant of variation for each equation. Then determine the slop fot he line that passes through each pair of points.

1.



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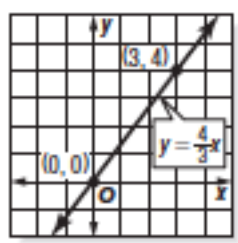


Write a direct variation equation that relates *x* and *y*. Assume that *y* varies directly as *x*. Then solve.

3. If $y = 7.5$ when $x = 0.5$, find y when $x = -0.3$.

4. If $y = 80$ when $x = 32$, find x when $y = 100$.

XV. Linear Equations in Slope-Intercept Form

Slope-Intercept Form

Slope-Intercept Form	$y = mx + b$, where m is the given slope and b is the y-intercept

Example 1 **Write an equation of the line whose slope is −4 and whose y-intercept is 3.**

$y = mx + b$ Slope-intercept form

$y = -4x + 3$ Replace m with −4 and b with 3.

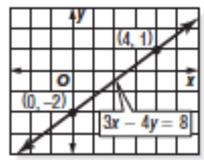
Example 2 **Graph $3x - 4y = 8$.**

$3x - 4y = 8$ Original equation

$-4y = -3x + 8$ Subtract $3x$ from each side.

$\frac{-4y}{-4} = \frac{-3x + 8}{-4}$ Divide each side by −4.

$y = \frac{3}{4}x - 2$ Simplify.



The y-intercept of $y = \frac{3}{4}x - 2$ is −2 and the slope is $\frac{3}{4}$. So graph the point (0, −2). From this point, move up 3 units and right 4 units. Draw a line passing through both points.

Practice:

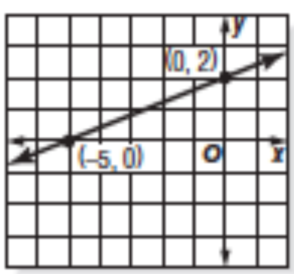
Write an equation of the line with the given slop and y-intercept.

1. slope: $\frac{1}{4}$, y-intercept: 3

2. slope: -2.5, y-intercept: 3.5

Write an equation of the line shown in each graph.

3.



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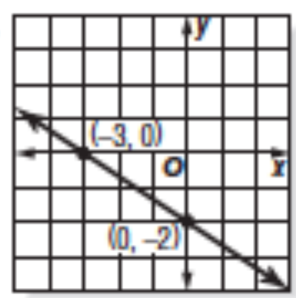
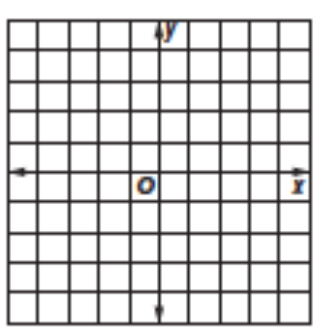
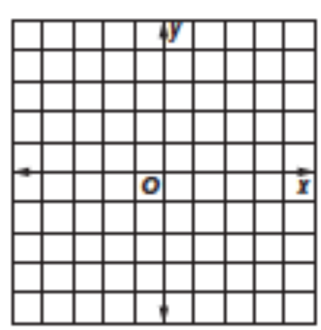


Graph each equation.

5. $y = -\frac{1}{2}x + 2$



6. $6x + 3y = 6$



XVI. Solving Word Problems

Translate each word problem into an algebraic equation, using x for the unknown, and solve. Write a **"let x ="** for each unknown; write an equation; solve the equation; substitute the value for x into the let statements(s) to answer the question.

For Example:

Kara is going to Maui on vacation. She paid $325 for her plane ticket and is spending $125 each night for the hotel. How many nights can she stay in Maui if she has $1200?

Step 1: What are you asked to fine? Let variables represent what you are asked to find.

How many nights can Kara stay in Maui?

Let x = The number of nights Kara can stay in Maui

Step 2: Write an equation to represent the relationship in the problem.

$325 + 125x = 1200$

Step 3: Solve the equation for the unknown

$$\begin{aligned} 325 + 125x &= 1200 \\ -325 \qquad &\quad -325 \\ 125x &= 875 \\ x &= 7 \end{aligned}$$

Kara can spend 7 nights in Maui

Practice: Write an algebraic equation to model each situation. Then solve the equation and answer the question.

1. A video store charges a one-time membership fee of $11.75 plus $1.50 per video rental. How many videos did Stewart rent if he spends $72.00?

2. Darel went to the mall and spent $41. He bought several t-shirts that ach cost $12 and he bought 1 pair of socks for $5. How many t-shirts did Darel buy?

3. Nick is 30 years less than 3 times Ray's age. If the sum of their ages is 74, how old are each of the men?

4. Three-fourths of the student body attended the pep-rally. If there were 1230 students at the pep rally, how many students are there in all?

5. Sarah drove 3 hours more than Michael on their trip to Texas. If the trip took 37 hours, how long did Sarah and Michael each drive?

6. Bicycle city makes custom bicycles. They charge $160 plus $80 for each day that it takes to build the bicycle. If you have $480 to spend on your new bicycle, how many days can it take Bicycle City to build the bike?